Publicity and the Clustering of IPO Underpricing*

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Abstract

We explain why underpricing in IPOs can be large in magnitude and clustered, using a signalling model where firms have private information about their qualities (high or low). A novel feature is that a firm, if perceived by the market as high quality, benefits from the industry's publicity which is an increasing function of the amount of IPO underpricing by all high-quality firms in the industry. Two separating equilibria exist, in one no firm underprices IPO and the industry has no publicity; in the other every high-quality firm underprices IPO and the industry has great publicity. The two equilibria coexist when the industry's publicity has a strong positive effect on each high-quality firm's expected earnings. A strong industry publicity induces underpricing because it increases both the benefit for a high-quality firm to signal its quality and the temptation for a low-quality firm to mimic; to benefit from the publicity, a high-quality firm underprices its IPO to separate itself from a low-quality firm. This result is opposite to a typical externality story where the free-rider problem would reduce or eliminate IPO underpricing altogether.

Keywords: Initial public offerings; Signalling; Externality; Multiple equilibria.
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1. Introduction

Initial public offerings (IPOs) by Internet firms experienced phenomenal price gains in the first half of 1999, implying huge underpricing in those IPO offer prices. For example, the share price of MarketWatch.com rose to $97.50 on the first trading day from an offer price of $17. In contrast, concurrent IPOs by traditional firms did not exhibit a general pattern of underpricing. In fact, in the first half of 1999 the IPO market was so concentrated on Internet firms and many of those IPOs performed so well that there was an increase in cancellations and withdraws from the IPO market by non-Internet firms.[1] Why do Internet firms offer their IPOs at prices so much below market prices? And, more importantly, why does underpricing cluster in time and industry? In this paper we construct a model to explain.

We argue that both the large magnitude and the clustering of IPO underpricing can be due to great uncertainty in the demand for Internet products or services. Since most Internet firms make products/services that have no resemblance to but nevertheless compete against traditional ones, there is very little guidance to predicting the demand for such products. Market expectations on those firms' earnings are sensitive to positive "publicity" that the Internet industry *as a whole* generates. A great publicity for the industry is likely to attract customers and improve market expectations on earnings for each good firm in the industry. We show that, through the industry's publicity, IPO underpricing can be clustered and large in magnitude.

More precisely, we model each firm's offer price and the fraction of shares issued to the public in IPO as devices to signal its quality. As in a typical signalling model (see later references), each firm's intrinsic quality (high or low) is known only to the firm itself and market expectations on a firm's earnings increase with the perceived quality of the firm. In contrast to a typical signalling model, a firm's expected earnings also depend on the industry's publicity, which is modeled as an increasing function of the amount of IPO underpricing by high-quality firms in the industry. A perceived high-quality firm's expected earnings increase with the industry's publicity by more than do a perceived low-quality firm's.

As in a typical signalling model, a low-quality firm tries to masquerade as high quality and so a high-quality firm can separate itself out only by taking actions that are too costly for a low-quality firm to take. This separation is possible here because the marginal cost of alternative financing methods such as borrowing is lower for a high-quality firm than for a low-quality firm. By restricting the amount of funds raised through IPO, a high-quality firm indicates that its

[1] As the chief executive of a large dry pet food company complained, "If you look at the IPO market, there's large-capitalization activity and dot.com activity, but little else. I feel sorry for small-cap companies that are nondot.com, and which need to complete their deals." (Prial, 1999)

quality is high to generate enough revenue to cover the resulted borrowing cost. Such signalling does not necessarily entail IPO underpricing; it does so only when the expected difference in expected earnings between a high-quality firm and a low-quality firm is sufficiently large. The industry's publicity induces underpricing by increasing such a difference.

There are two separating equilibria in this framework.[2] In one equilibrium there is large underpricing of IPOs by all high-quality firms, thus clustering, but in the other there is no underpricing. The no-underpricing equilibrium is the only equilibrium when the influence of the industry's publicity on each firm's expected earnings is weak. But when such influence is strong, the two equilibria coexist. The coexistence of the equilibria is due to self-fulfilling expectations. When a high-quality firm expects the industry's publicity to be high, the difference in expected earnings between a high-quality firm and a low-quality firm is great. Each individual high-quality firm wants to benefit from the publicity by underpricing its own IPO, thereby creating the clustering in underpricing which fulfills the expectations of a high industry publicity. In contrast, when the industry's publicity is expected to be low, the benefit from underpricing one's own IPO is small and so no individual firm wants to underprice its IPO, supporting the expectations of a low industry publicity. In this case, high-quality firms signal instead by offering fewer shares to the public at the full price.

The underpricing equilibrium entails large underpricing. When all high-quality firms are expected to underprice, the benefit from the industry's publicity is so large that makes low-quality firms very likely to mimic high-quality firms; to signal successfully a high-quality firm must incur a sufficiently high cost in the form of great underpricing. In a simple version of the model, a high-quality firm gives some shares free to the public in the underpricing equilibrium.

It should be emphasized that high-quality firms underprice IPOs here in order to benefit from the industry's publicity rather than build one's own, although we also examine the latter motive. Given that the industry's publicity is a public good, the positive link established here between the clustering of IPO underpricing and the industry's publicity is opposite to a typical externality story. In usual circumstances, each high-quality firm wants to enjoy the publicity that other firms' create through IPO underpricing but is reluctant to underprice its own (since IPO underpricing is costly). What overcomes this free-rider problem in our framework is the presence of private information about firms' qualities. Since a firm can benefit from the industry's publicity only when it can convince the market of its high quality, a greater industry publicity makes each high-quality firm more willing to signal. At the same time a greater industry publicity makes

[2] All equilibria we focus on in this paper are separating equilibria that are refined by the intuitive criterion of Cho and Kreps (1987). For each value of expected earnings, there is a unique separating equilibrium in the signalling game, but there are multiple values of expected earnings that are consistent with rational expectations.

signalling more difficult by increasing low-quality firms' temptation to mimic, therefore making IPO underpricing a necessary action for successfully signalling quality.[3]

There is a large literature on IPO underpricing. Pioneering examples of signalling models include Allen and Faulhaber (1989), Welch (1989) and Grinblatt and Hwang (1989), who focus on a single firm's signalling decisions. The main contribution of the current paper to this literature is to show that large underpricing can be clustered in time and industry. In addition, our paper recovers a U-shaped relationship between the fraction of shares issued to the public in IPO and expected earnings. When expected earnings increase from a low level, a high-quality firm signals the quality by reducing the number of shares issued to the public, without underpricing IPO. When expected earnings increase from a high level, the firm signals by underpricing IPO and increasing the number of shares issued to the public. This result reconciles the IPO signalling literature with the lack of a monotonic relationship between the IPO price and the fraction of shares withheld by firms' original owners (Michaely and Shaw (1994)).[4]

There are other theories that explain IPO underpricing. For example, Rock (1986) argues that underpricing is an outcome of the winner's curse, since uninformed investors must be compensated with a low offer price in order to participate. Others attribute a role to underwriters in IPOs (e.g., Beatty and Ritter (1986) and Benveniste and Spindt (1989)). Although these alternative explanations are useful and do not necessarily contradict the signalling story, it is not clear how they imply the clustering of IPO underpricing. For the focus of this paper, we will abstract from any role of underwriters. The emphasis on a firm's own desire to underprice underscores the rationality of such underpricing.[5]

The remainder of this paper is organized as follows. Section 2 describes the demand uncertainty facing Internet firms and the signalling game in IPO. Section 3 solves for the signalling equilibrium, taking as given the earning difference between a high-quality and a low-quality firm. Section 4 solves for market equilibria where expected earnings depend on the amount of underpricing in the industry. Section 5 explores several extensions. Section 6 concludes the paper and the appendix provides necessary proofs.

[3]In the current model, clustering occurs as firms try to signal their qualities. This is different from the so-called "herd" behavior (Banerjee (1992) and Bikhchandani, et al (1992)), where agents ignore their private information and follow previous agents' actions. An apparent difference between the two types of behavior is that herding occurs only when firms move sequentially but clustering can occur when firms move simultaneously.

[4]The role of alternative financing methods such as borrowing also links our model generally to Myers and Majluf (1984) and specifically to James and Wier (1990) and Slovin and Young (1990). The latter two papers have shown that IPOs of firms with previously established borrowing relationships are underpriced less than other IPOs. We do not examine this use of previous borrowing records to signal but rather the reverse casaulity: Underpricing IPO forces a firm to use alternative financing methods.

[5]Despite the huge underpricing, Internet firms do not seem to lay blames on their underwriters. As one chief executive officer of a newly public Internet firm put it, "We don't second-guess what we left on the table. Our eyes are on the future in terms of building a great company." (Smith and Simon, 1999).

2. The Structure of the Model

Consider an industry with $n \geq 2$ risk-neutral firms, each having a project that requires external financing of an amount normalized to one. The project's quality, denoted x, is either x_H (high-quality) or x_L (low-quality), where $x_H > x_L > 0$. The precise value of x is known only to the firm. The public has the prior belief that $x = x_H$ with probability α and $x = x_L$ with probability $1-\alpha$, where $\alpha \in (0,1)$. To simplify analysis, let us assume that all n firms decide to seek financing at the same time (see Section 5 for discussions on sequential decisions).

A firm can raise the required amount by initial public offering (IPO) of its equity. Let the total number of shares of a firm be 1. The firm chooses the number of shares to be issued to the public in IPO, $f \in (0,1]$, and the offer price s. The original owners of the firm keep $1-f$ shares. The market price is denoted p. The shares are said to be underpriced if $s < p$. Denote the amount of underpricing by $d \equiv p - s$. The amount of fund raised through IPO is $q \equiv sf$.

If $q < 1$, the remainder of the investment is obtained through alternative financing methods. Although there may be many such alternative methods, we will simply refer to them as borrowing. The total cost of borrowing is $\left(1 + bx^{-1}\right)(1-q)$ for $q < 1$, where $b > 0$ is a constant. Thus, for each dollar borrowed, there is an additional cost bx^{-1}. An important feature is that the borrowing cost is a decreasing function of the project's quality. This is realistic: Although the project's quality is unknown to the public, the lenders can screen the project to find the true quality with positive probability and hence offer a lower loan rate to high-quality projects. Note also that, for simplicity, the borrowing cost is assumed to be linear in the amount of borrowing.[6]

Let a firm's earning be r_H if it is high-quality and r_L if it is low-quality. There are two components in earnings. The first component depends on the project's quality and is $R_0 x_i / x_L$ for the project with quality $i = H, L$, where $R_0 > 0$. This component is known to the firm and is known to the market once the project's quality is revealed. We refer to this component as *intrinsic earnings* of the firm. The second component of the earnings is uncertain to the firm and hence uncertain to the market even when the project's quality is observed. Let this component be m for a high-quality firm and, to simplify, 0 for a low-quality firm. Then, the earnings for a low-quality firm and a high-quality firm are:

$$r_L = R_0, \qquad r_H = R_0 \frac{x_H}{x_L} + m. \tag{2.1}$$

The uncertain component m is meant to capture the uncertainty in the demand for products in a new industry, such as the Internet industry. Most Internet firms produce products or services

[6] All analytical results in this paper hold also for the cost function $(1+b/x)C(1-q)$ with the properties $C(0) = 0$, $C'(0) \geq 1$ and $C'' > 0$.

that have no resemblance to traditional ones, but they nevertheless compete against traditional sectors for customers.[7] Public awareness of the industry is important to the growth of each firm in the industry. Spectacular price gains in IPOs create such publicity. Moreover, a high-quality firm benefits more from the industry's publicity than does a low-quality firm and so the expected value of m is positive.

To focus on the positive externality created by the industry's publicity, we abstract from the competition among firms in the industry (see Section 6 for a discussion). For the moment we also abstract from the effect a firm's IPO underpricing has on its own expected earnings (Section 5 analyzes this effect) and assume that market expectations on a firm's m depend only on *other* firms' underpricing. Precisely, let D be the amount of underpricing in IPO by a representative firm that is perceived as high quality; if the market perceives a firm to be high quality, the expectations on m conditional on D are

$$E(m|D, \text{the firm's perceived quality} = x_H) = \rho D, \tag{2.2}$$

where $\rho \in (0,1)$ is a constant. We assume that n is a large number so that a high-quality firm's own expectations on m at the time of deciding the offer price, conditional on D, are arbitrarily close to the market's expectation.[8]

Market expectations of a perceived high-quality firm are $R_H \equiv R_0 x_H / x_L + \rho D$. Since each individual firm takes D as given, R_H is exogenous to each firm. Let I be the probability with which the market believes that a firm is high-quality, after observing all n firms' IPO prices. Then the firm's earning expected by the market is

$$E(r|I) = R_I \equiv I \cdot R_H + (1-I)R_0. \tag{2.3}$$

The game is played as follows. First, each firm simultaneously chooses and commits to offering f shares of its equity at an offer price s, while taking R_H as given. After IPO the firm borrows if the IPO receipts are not sufficient to cover the project's investment. Finally, the project is carried out and the earnings are realized. Each firm pays the creditors first if there is any debt and then to the shareholders. Notice that at the time of IPO the firm's borrowing cost is not

[7]For example, selling books, auctioning goods, providing market information on Internet compete directly against businesses that organize such activities in traditional ways.

[8]This assumption is made for simplicity. When a firm makes the IPO decision, it does not observe other firms' IPO decisions and so its expectations of its own m are $\rho D[1-(1-\alpha)^{n-1}]$, where $1-(1-\alpha)^{n-1}$ is the probability that there is at least one other high-quality firm coming to the IPO market. Such expectations are different from market expectations (which are made after observing all firms' IPO decisions) and the discrepancy by itself can make the offer price deviate from the market price. This discrepancy vanishes when n is large or when firms move sequentially (Section 5).

Also, one can specify D as the average of underpricing by other high-quality firms rather than the amount of underpricing by a representative high-quality firm as is done here. The analytical results will not change.

publicly observed since the borrowing has not yet occurred, although the amount of borrowing $1-q$ can be inferred. We also assume that the net risk-free rate is zero and that the product market customers have the same information as the investors.

Let us isolate an arbitrary firm and examines its decision. It is convenient to express the firm's decisions as $a \equiv (f,q)$ rather than (f,s). In choosing a, the firm intends to maximize the expected return to the original owners, which is

$$V(f,q;R_I,x) \equiv (1-f)\left[R_I - \left(1+bx^{-1}\right)(1-q)\right]. \tag{2.4}$$

This return is known to the firm, since the firm's quality x is known to the firm itself.[9]

To investors, what matters is the expected return from holding the firm's shares, which is

$$R_I - \left(1+bE_Ix^{-1}\right)(1-q),$$

where $E_Ix^{-1} = Ix_H^{-1} + (1-I)x_L^{-1}$. Let p_I be the market price of a firm's share when the market belief is I. Under rational expectations, the market price equals the expected return to shareholders and so the expected rate of return per share equals the risk-free rate. Also, for investors to participate in IPO, the offer price cannot exceed the market price. Thus

$$0 \le s = q/f \le p_I = R_I - \left(1+bE_Ix^{-1}\right)(1-q). \tag{2.5}$$

As is standard in this environment, a high-quality firm may want to signal its quality. One way to signal is to reduce the amount of funds raised through IPO. Since any amount that is not financed through equity must be obtained through alternative financing methods which bear an additional cost, reducing the fund raised through IPO signals that the firm's earning ability might be sufficiently high to justify such a cost. Signalling can be done through the offer price and/or the number of shares issued to the public in IPO. Since all firms want the market to believe that they are high quality, a low-quality firm may want to mimic a high-quality firm. A necessary condition for successful signalling by a high-quality firm is that it has a higher incentive or ability to signal than does a low-quality firm. This is the well-known single-crossing property, satisfied in the current model in the following forms:

$$\frac{\partial}{\partial x}\left[-\frac{\partial V(f,q;R,x)}{\partial R}\bigg/\frac{\partial V(f,q;R,x)}{\partial q}\right] = \frac{\partial}{\partial x}\left[-\frac{1}{1+bx^{-1}}\right] < 0; \tag{2.6}$$

$$\frac{\partial}{\partial x}\left[-\frac{\partial V(f,q;R,x)}{\partial R}\bigg/\frac{\partial V(f,q;R,x)}{\partial f}\right] = \frac{\partial}{\partial x}\left[\frac{1-f}{R-(1+bx^{-1})(1-q)}\right] < 0. \tag{2.7}$$

[9] Throughout this paper the payoff to a firm refers to the payoff to the original owners of the firm after IPO, not that to all shareholders.

These properties are illustrated in Figures 1a and 1b. The relation (2.6) states that, to receive the same increase in the expectations on earnings "rewarded" by the market, a high-quality firm is willing to reduce the fund raised through IPO by more than does a low-quality firm. Since the number of shares issued to the public is held fixed in (2.6), the relation equivalently states that a high-quality firm is willing to reduce the offer price by more than does a low-quality firm for the same reward in expected earnings. (2.7) states that, for fixed IPO receipts, a high-quality firm is willing to increase the number of shares issued to the public by more than does a low-quality firm in order to receive the same reward in expected earnings. Since the IPO receipts are held fixed in (2.7), the relation again states that a high-quality firm is more willing to underprice its IPO than a low-quality firm. Both properties come directly from the assumption that a high-quality firm faces a lower borrowing cost than a low-quality firm.

Figures 1a and 1b here.

To focus on interesting cases, we now narrow our attention:

Assumption 1. 1A. *A high-quality firm, if its quality is publicly known, can make a positive return even when the investment is* 100% *debt financed, i.e.,*

$$R_0 \frac{x_H}{x_L} - \left(1 + \frac{b}{x_H}\right) > 0.$$

1B. *A low-quality firm, if its quality is publicly known, cannot make a positive return when the investment is* 100% *debt financed, i.e.,*

$$R_0 - \left(1 + \frac{b}{x_L}\right) < 0.$$

1C. *A low-quality firm has a positive payoff if the investment is* 100% *equity financed, even when the firm's quality is publicly known. That is,* $V(f, 1; R_0, x_L) > 0$ *for some* $f = 1/s \geq 1/p_L$.
1D. *The intrinsic earning difference between high-quality and low-quality firms is not too large:*

$$R_0 \left(\frac{x_H}{x_L} - 1\right) < \frac{b}{x_L}.$$

Assumption 1A provides a high-quality firm with an incentive to signal its quality: Since it makes a positive return even with 100% debt financing, it can reduce the IPO receipt to signal its high quality. The signalling attempt may or may not require underpricing of IPO. Assumptions 1B and 1C make it desirable for a low-quality firm to finance its investment through equity if its quality is publicly known. Since the quality is not publicly known, these assumptions do not preclude a low-quality firm from using debt financing to mimic a high-quality firm. Assumption

$1D$ is used to isolate the importance of externality in firms' decisions to underprice IPO: In absence of the externality, Assumption $1D$ ensures that there is no underpricing (see Section 4).

Assumption $1C$ can be simplified. To do so, we obtain the maximum payoff to a low-quality firm when the firm's quality is publicly known. With known low quality, the market price of the firm's share is $p_L = R_0 - \left(1 + bx_L^{-1}\right)(1-q)$. Since the number of shares issued in the IPO must satisfy $f \geq q/p_L$, the payoff to a known low-quality firm satisfies

$$\begin{aligned} V(f, q; R_0, x_L) &\leq \left(1 - \tfrac{q}{p_L}\right)\left[R_0 - \left(1 + \tfrac{b}{x_L}\right)(1-q)\right] \\ &= R_0 - \left(1 + \tfrac{b}{x_L}\right)(1-q) - q. \end{aligned}$$

The last expression is increasing in q and so it is maximized at $q = 1$, generating a value $R_0 - 1$. Conversely, if a low-quality firm chooses to reveal its low quality, it can always choose the actions $(q, f) = (1, 1/R_0)$ and obtain the payoff $R_0 - 1$. Thus, Assumption $1C$ can be replaced by:

$1C'$. $R_0 > 1$.

Before getting into the details of the signalling game, it is important to note that the externality *per se* does not generate clustering of underpricing, as stated below:

Proposition 2.1. *If firms' qualities are public information then there is no underpricing in equilibrium.*

When each firm's quality is public knowledge, there is no need for a high-quality firm to signal its quality and a low-quality firm cannot masquerade as a high-quality firm. Each firm's payoff is maximized by setting q to 1 and the offer price to the corresponding market price. Each high-quality firm wants to benefit from other high-quality firms' underpricing but is *unwilling* to underprice its own IPO. This free-rider problem ensures that in equilibrium with public information there is no underpricing by any firm. The existence of private information is critical for overcoming this free-rider problem.

3. Signalling Equilibrium

For arbitrarily given R_H that satisfies Assumptions $1A - 1D$, we characterize a firm's strategies. The result is a best response by the firm to other firms' strategies. For lack of an appropriate term, we refer to this single firm's best response together with the market's belief as a signalling equilibrium. In contrast, the true equilibrium where R_H is also determined is called a market equilibrium, an object examined in the next section.

A Bayesian perfect signalling equilibrium is such that for any given R_H that satisfies Assumptions $1A - 1D$, the following conditions hold: (i) The choices (f, q) maximize the firm's payoff

$V(f, q; R_I, x)$ given the beliefs; and (ii) The beliefs are rational according to Bayes updating given the firm's choices. As is well known, this definition does not sufficiently limit the scope of equilibrium since the beliefs off the equilibrium path are arbitrary. In this paper we will employ the intuitive criterion by Cho and Kreps (1987) to refine the equilibrium (see below) and throughout this paper the term "equilibrium" means an equilibrium that satisfies this criterion.

Two types of signalling equilibria may exist. One is pooling equilibria in which both high-quality and low-quality firms take the same action; the other is separating equilibria in which the two types of firms take different actions and market's beliefs sort them out according to their actions. Although separating equilibria are the interest of our analysis, oddly enough the best way to describe them is to describe a pooling equilibrium first.

Suppose that the two types of firms take the same action $a_0 \equiv (f_0, q_0)$ in a pooling equilibrium. Then the market's belief upon observing a_0 is that the firm is high-quality with probability $I = \alpha$. Denote the payoff to a high-quality firm in this pooling equilibrium by $V^0(x_H) \equiv V^0(f_0, q_0; R_\alpha, x_H)$. Similarly denote $V^0(x_L) \equiv V(f_0, q_0; R_\alpha, x_L)$ as the payoff to a low-quality firm. For the action and the belief to form a pooling equilibrium, the following necessary (but not sufficient) conditions must be met:

$$f_0, q_0 \in [0, 1]; \tag{3.1}$$

$$q_0/f_0 \le p_\alpha = R_\alpha - \left(1 + bE_\alpha x^{-1}\right)(1 - q); \tag{3.2}$$

$$V^0(x_L) \ge R_0 - 1. \tag{3.3}$$

The first condition is self-explanatory; the second condition requires the offer price to be at most the market price; the last condition requires a low-quality firm's payoff in the pooling equilibrium to be at least that from revealing the firm's type and choosing $(q, f) = (1, 1/R_0)$. This condition also implies that a high-quality firm gets a higher payoff in the pooling equilibrium than from choosing $(q, f) = (1, 1/R_0)$ and being viewed by the market as a low-quality firm.

The conditions (3.1) and (3.2) can be written in more useful forms. First, since $q_0/f_0 \le p_\alpha$, the restriction $f_0 \in [0, 1]$ is equivalent to $p_\alpha \ge q_0 \ge 0$. With the expression of price in (3.2), (3.1) can be rewritten as

$$1 \ge q_0 \ge Q_0 \equiv \max\left\{0, 1 - \frac{R_\alpha - 1}{bE_\alpha x^{-1}}\right\}. \tag{3.4}$$

Since $R_\alpha > R_0 > 1$ by Assumption $1C'$, $Q_0 < 1$. For $q_0 \ge Q_0$, (3.2) can be replaced by

$$f_0 \ge S_\alpha(q_0) \equiv q_0 / \left[R_\alpha - \left(1 + bE_\alpha x^{-1}\right)(1 - q_0)\right]. \tag{3.5}$$

Also, the payoffs $V^0(x_H)$ and $V^0(x_L)$ are bounded above by $R_\alpha - 1$. Intuitively, the amount $R_\alpha - 1$ is the firm's payoff when the firm obtains the expected earnings R_α without incurring any borrowing cost. This is the best the original owners can get in a pooling equilibrium, since the pooling equilibrium may involve less than 100% equity financing. This intuition is stated below (see Appendix A for a proof).

Lemma 3.1. $V^0(x_H), V^0(x_L) \leq R_\alpha - 1$.

In the set of actions that satisfy (3.3), (3.4) and (3.5), only those that do not leave any room for "credible" deviations by a high-quality firm satisfy the Cho-Kreps intuitive criterion. To describe a credible deviation by a high-quality firm, suppose a deviation $(f, q) \neq (f_0, q_0)$ satisfies the following conditions. First, the deviation is feasible for a high-quality firm, i.e., $f, q \in [0, 1]$ and the offer price does not exceed the implied market price:

$$0 \leq q/f \leq p_H = R_H - (1 + bx_H^{-1})(1 - q). \tag{3.6}$$

Second, the deviation generates a lower payoff to a low-quality firm than in the pooling equilibrium, even when the firm is viewed as a high-quality firm as a result of the deviation:

$$(1 - f)\left[R_H - (1 + bx_L^{-1})(1 - q)\right] < V^0(x_L). \tag{3.7}$$

Third, the deviation generates a higher payoff to the high-quality firm than in the pooling equilibrium when the firm is viewed as a high-quality firm as a result of the deviation:

$$(1 - f)\left[R_H - (1 + bx_H^{-1})(1 - q)\right] > V^0(x_H). \tag{3.8}$$

Actions (f, q) that satisfy (3.6), (3.7) and (3.8) are credible deviations by a high-quality firm.

The credible deviations can be understood as follows. Deviations that satisfy (3.6) and (3.7) are feasible to the firms but yield lower payoffs to a low-quality firm than in the pooling equilibrium, even when the deviator is given the benefit of doubt and viewed as a high-quality firm. Thus, a low-quality firm will not make such deviations. If the deviations also satisfy (3.8), a high-quality firm would want to make such deviations, given that the market views the deviator as a high-quality firm. Thus, observing deviations that satisfy (3.6) – (3.8), the market should intuitively interpret the deviator as a high-quality firm. To satisfy this intuitive criterion, a pooling equilibrium cannot allow for deviations that satisfies (3.6)–(3.8). This restriction on beliefs off the equilibrium path eliminates a plethora of equilibria.[10]

Let us first examine the set of actions that satisfy (3.6) and (3.7). Under Assumption $1A$, (3.6) can be rewritten as $f, q \in [0, 1]$ and

$$f \geq S_H(q) \equiv q/\left[R_H - (1 + bx_H^{-1})(1 - q)\right]. \tag{3.9}$$

[10] In the current context, separating equilibria that satisfy the intuitive criterion are the Riley (1979) outcomes.

To rewrite (3.7), define a critical level of q as

$$Q_1 \equiv 1 - \frac{R_H - V^0(x_L)}{1 + bx_L^{-1}}. \tag{3.10}$$

Note that Q_1 is less than one but is not necessarily greater than zero, depending on the magnitude of R_H. If either $Q_1 < 0$ or $q \leq Q_1$ then (3.7) is satisfied for all $f \in [0, 1]$. For $q \geq \max\{0, Q_1\}$, (3.7) can be rewritten as

$$f > IND_L(q) \equiv 1 - V^0(x_L) / \left[R_H - (1 + bx_L^{-1})(1 - q) \right]. \tag{3.11}$$

The notation IND_L indicates that a low-quality firm is indifferent between choosing the pooling action (f_0, q_0) and being viewed as a high-quality firm by choosing actions satisfying $f = IND_L(q)$. The set of actions that satisfy the above restrictions is the shaded area in Figure 2a for $Q_1 < 0$ and in Figure 2b for $Q_1 > 0$.

Figures 2a and 2b here.

The following lemma formally states some properties of the two curves $S_H(q)$ and $IND_L(q)$ in Figures 2a and 2b (see Appendix A for a proof):

Lemma 3.2. *Under Assumptions* $1A - 1C$, $IND_L(q)$ *is an increasing and concave function for all* $q > Q_1$. $S_H(q)$ *is an increasing and concave function for all* $q > 0$. *If* $Q_1 < 0$, *then* $IND_L(q) > S_H(q)$ *for all* $q \geq 0$; *If* $Q_1 \geq 0$, *then there is a unique solution to* $IND_L(q) = S_H(q)$ *in the range* $q \geq Q_1$, *denoted* Q_A, *and* $IND_L(q) > S_H(q)$ *if and only if* $q > Q_A$. *Furthermore, a high-quality firm's payoff is an increasing function of* q *along* $f = S_H(q)$ *and a decreasing function of* q *along* $f = IND_L(q)$.

A high-quality firm can consider a deviation from the supposed pooling equilibrium to actions in the shaded areas in Figures 2a and 2b. As argued before, the market should intuitively view such deviations as coming from a high-quality firm and attach a belief $I = 1$ to the deviation. By the same token, a high-quality firm should consider only deviations that maximize its payoff. That is, any deviations in the shaded area that are not the best cannot be an equilibrium that satisfies the Cho-Kreps criterion, since further deviations from those actions to the best actions do not change the market's belief ($I = 1$) but increase the firm's payoff.

The best deviation by a high-quality firm from the supposed pooling equilibrium is arbitrarily close to and above the action depicted by point A in Figure 2a if $Q_1 < 0$ and in Figure 2b if $Q_1 > 0$. To see this, note that the firm's payoff increases when actions move toward the southeast in Figures 2a and 2b and so the best deviations are located arbitrarily close to and above the lower boundaries of the shaded areas in these figures. Moreover, since a high-quality firm's payoff is an increasing function of q along $f = S_H(q)$ and a decreasing function of q along $f = IND_L(q)$ (see Lemma 3.2), the best deviation is arbitrarily close to and above point A in Figures $2a$ (or $2b$). The limit of this deviation is point A.[11] That is, the best deviation is

$$(f_b, q_b) \equiv \begin{cases} \left(1 - \frac{V^0(x_L)}{R_H - 1 - bx_L^{-1}}, \quad 0\right), & \text{if } Q_1 \leq 0 \\ \\ (S_H(Q_A), \quad Q_A), & \text{if } Q_1 > 0. \end{cases} \tag{3.12}$$

We now check whether this deviation increases the payoff to a high-quality firm. Relative to the pooling equilibrium, the deviation (f_b, q_b) generates the following gain to a high-quality firm:

$$\begin{aligned} & (1 - f_b)\left[R_H - (1 + bx_H^{-1})(1 - q_b)\right] - V^0(x_H) \\ = \; & (1 - f_b)\left[R_H - (1 + bx_H^{-1})(1 - q_b)\right] - (1 - f_b)\left[R_H - (1 + bx_L^{-1})(1 - q_b)\right] \\ & + \left\{(1 - f_b)\left[R_H - (1 + bx_L^{-1})(1 - q_b)\right] - V^0(x_L)\right\} + \left[V^0(x_L) - V^0(x_H)\right] \\ = \; & b(x_L^{-1} - x_H^{-1})(1 - f_b)(1 - q_b) - b(x_L^{-1} - x_H^{-1})(1 - f_0)(1 - q_0) \\ = \; & \tfrac{b(x_L^{-1} - x_H^{-1})}{1 + bx_L^{-1}}\left[(1 - f_b)R_H - (1 - f_0)R_\alpha\right]. \end{aligned}$$

The first equality follows from adding and subtracting the same terms; the second equality follows from the fact that the term in $\{.\}$ is zero by the definitions of (f_b, q_b); the third equality follows from substituting the definitions of q_b and q_0.

The payoff to a high-quality firm from the deviation to (f_b, q_b) is greater than in any pooling equilibrium iff $(1 - f_b)R_H > (1 - f_0)R_\alpha$. In this case, there is no pooling equilibrium (that satisfies the Cho-Kreps intuitive criterion) and so the best action for a low-quality firm is $(f, q) = (1/R_0, 1)$, yielding a payoff $R_0 - 1$. Replacing $V^0(x_L)$ by $R_0 - 1$, the condition $Q_1 \leq 0$ becomes $R_H \geq R_0 + bx_L^{-1}$ and we denote the corresponding values of (f_b, q_b) by (f^*, q^*). That is, for $R_H \geq R_0 + bx_L^{-1}$,

$$f^* = 1 - \frac{R_0 - 1}{R_H - (1 + bx_L^{-1})}, \qquad q^* = 0; \tag{3.13}$$

and for $R_H < R_0 + bx_L^{-1}$,

$$\begin{cases} f^* = S_H(q^*); \\ \\ \frac{q^*}{R_H - (1 + bx_H^{-1})(1 - q^*)} = 1 - \frac{R_0 - 1}{R_H - (1 + bx_L^{-1})(1 - q^*)}. \end{cases} \tag{3.14}$$

We have the following propositions (see Appendix B for a proof):

[11] In the borderline case $Q_1 = 0$ (where point A coincides with the origin of the plane), the best deviation is $f = \varepsilon > 0$ and $q = 0$, where ε is sufficiently small. Since this case involves underpricing, it can be grouped with the case $Q_1 < 0$.

Proposition 3.3. *Under the Cho-Kreps intuitive criterion, there is a unique separating signalling equilibrium for given R_H. In such an equilibrium, a high-quality firm's actions are (f^*, q^*), characterized by (3.13) when $R_H - R_0 \geq bx_L^{-1}$ and by (3.14) when $R_H - R_0 < bx_L^{-1}$. A low-quality firm's actions are $(f, q) = (1/R_0, 1)$.*

Proposition 3.4. *A pooling signalling equilibrium for given R_H exists only when $Q_1 < 0$ and $q_0 < 1 - R_\alpha/R_H$. There exist $\bar{\alpha}, \underline{\alpha} \in (0, 1)$ such that, if $\alpha > \bar{\alpha}$, pooling equilibria exist for suitably restricted actions and beliefs. If $\alpha < \underline{\alpha}$, no pooling equilibrium exists.*

The reason that some pooling equilibria survive the Cho-Kreps refinement is that the amount of funds raised through IPO cannot be less than zero. This limits the extent to which a high-quality firm can signal. When the expected earning of a high-quality firm is sufficiently higher than that of a low-quality firm, the high-quality firm must incur a sufficiently high signalling cost in order to prevent a low-quality from mimicking. With the lower bound on the amount of fund raised through IPO, this becomes difficult and so some pooling equilibria with small IPO receipts can survive. In particular, when the prior for a high-quality firm, α, is large, the difference between R_α and R_0 is large and so the benefit from mimicking is large. Even with a very low q, a low-quality firm may get a higher payoff from mimicking than from taking the separating action, in which case the pooling equilibrium exists. When α is small, in contrast, mimicking does not pay and so the pooling equilibrium does not exist.

A high-quality firm has a preference over the two ways to signal. One is to reduce the number of shares issued to the public in IPO without underpricing and the other is to underprice IPO. Both methods reduce the amount of funds raised through IPO and achieve the purpose of signalling. However, reducing the number of shares without underpricing IPO is preferable. By reducing the number of shares issued to the public without underpricing, the firm keeps a larger stake of the firm and hence of its future earnings. In contrast, if the firm underprices its IPO it must give up a large number of shares to the public in IPO in order to raise the same amount of funds. This is more costly to the original owners of the firm than the first method since they give up a larger claim on the firm's future earnings.

Despite this preference, a high-quality firm chooses to underprice IPO in some cases. This is because there is a limit to which a high-quality firm can signal by reducing f . Even reducing f to zero can only signal an expected earning of $R_0 + bx_L^{-1}$.[12] For expected earnings that are higher than this level, the firm must sacrifice even more in order to prevent a low-quality firm from mimicking and this requires underpricing IPO.

[12] This is obtained by setting $(f, q) = (0, 0)$ and $V^0(x_L) = R_0 - 1$ in the equality form of (3.11).

Therefore, the number of shares issued by the firm to the public in IPO has a U-shaped relationship with the firm's expected earnings, as depicted in Figure 3. When a high-quality firm's expected earnings increase from low levels, the number of shares issued to the public decreases, while IPO is at the full market price. This continues until the number of shares issued to the public reaches a minimum (which is zero in this version of the model). When expected earnings increase further, there is no more room to cut the number of shares issued to the public and so the firm signals by discounting IPO and increasing f. When the minimum of f is above zero, as shown in Section 5, the response of f to expected earnings can still be U-shaped even in the underpricing region. This absence of a monotonic (negative) relationship between f and underpricing is in contrast to previous signalling models but is consistent with the empirical finding of Michaely and Shaw (1994).

Figures 3 and 4 here.

A high-quality firm's best response to other high-quality firms' decisions can be summarized by an underpricing curve, depicted in Figure 4. Denote

$$D_0 = \frac{1}{\rho}\left[\frac{b}{x_L} - R_0\left(\frac{x_H}{x_L} - 1\right)\right]. \tag{3.15}$$

Then $R_H - R_0 \geq b x_L^{-1}$ if and only if $D \geq D_0$. Note that Assumption 1D requires $D_0 > 0$. A high-quality firm's underpricing is

$$d = \begin{cases} 0, & \text{if } D < D_0 \\ p_H = \rho D + R_0 x_H / x_L - 1 - b x_H^{-1}, & \text{if } D \geq D_0. \end{cases} \tag{3.16}$$

4. Market Equilibrium

Now we solve for the market equilibrium by solving for the expected earnings R_H. From now on only the separating equilibrium is considered. Since such equilibrium is unique for any given R_H, the multiplicity of market equilibria in this section has nothing to do with the usual multiplicity associated with signalling equilibria; instead, it arises because multiple values of R_H can be supported by rational expectations.

A symmetric market equilibrium is a pair (d, D) such that $d = D$ and that d is a best response to D given by (3.16). Imposing the symmetric requirement $d = D$ on (3.16) solves for the market equilibrium, as shown in Figure 4. To characterize market equilibria, denote

$$\underline{\rho} \equiv \frac{b x_L^{-1} - R_0(x_H/x_L - 1)}{b(x_L^{-1} - x_H^{-1}) + R_0 - 1}. \tag{4.1}$$

Note that $0 < \underline{\rho} < 1$ under Assumptions $1A$ and $1D$. The following proposition can be directly proved and the proof is omitted:

Proposition 4.1. *Under Assumptions $1A-1D$, a market equilibrium with no IPO underpricing exists for all $0 \leq \rho < 1$. A market equilibrium with IPO underpricing exists if and only if $\underline{\rho} \leq \rho < 1$. Thus, when $0 \leq \rho < \underline{\rho}$, only the no-underpricing market equilibrium exists; when $\underline{\rho} \leq \rho < 1$, both the underpricing equilibrium and the no-underpricing equilibrium exist.*

Figure 4 depicts the case $\underline{\rho} < \rho < 1$. The no-underpricing market equilibrium is at point EN and the underpricing market equilibrium is at point EU.[13]

Several aspects of the above proposition are noteworthy. First, the no-underpricing equilibrium exists for all $\rho \in [0, 1)$. It is the only equilibrium when $0 \leq \rho < \underline{\rho}$, in which case the level D_0 is sufficiently large and the underpricing curve is sufficiently flat that the entire underpricing curve lies below the 45-degree line in Figure 4. An implication of this result is that there would be no IPO underpricing if there were no externality through the industry's publicity.[14]

Second, the underpricing equilibrium and the no-underpricing equilibrium both exist when the externality is sufficiently strong (i.e. when $\underline{\rho} \leq \rho < 1$). The coexistence of the two types of equilibria is an outcome of self-fulfilling expectations. If a high-quality firm expects that other high-quality firms will not underprice, the difference in expected earnings between a high-quality firm and a low-quality firm is not large, as maintained by Assumption $1D$. Then the low-quality firm's temptation to mimic is not very strong, in which case a high-quality firm can separate itself from a low-quality firm by reducing the number of shares in IPO without underpricing. In contrast, if a high-quality firm expects that other high-quality firms will underprice, the difference in expected earnings between a high-quality firm and a low-quality firm is large, due to the industry's publicity. A low-quality firm's temptation to mimic is strong in this case and so underpricing is necessary for a high-quality firm to separate itself from a low-quality firm. The coexistence of a no-underpricing equilibrium with the underpricing equilibrium illustrates the fragility of large underpricing in IPOs.

Third, the large underpricing tends to be clustered in time and in particular industries, since underpricing is the best response to other firms' underpricing in the underpricing equilibrium. As noted before (Proposition 2.1), the externality through the industry's publicity is necessary

[13]When $\underline{\rho} \leq \rho < 1$, there might also be a mixed-strategy equilibrium at $D = D_0$ if we allow each high-quality firm to underprice with a probability in $(0, 1)$. To the issues that we focus on in this paper, such as clustering of underpricing, examining the mixed-strategy equilibrium does not add much.

[14]As noted before, Assumption $1D$ is important for this result. When the intrinsic earning difference between a high-quality and a low-quality firm is large enough to violate Assumption $1D$, then $\underline{\rho} < 0$ and there is a need for a high-quality firm to underprice anyway. In fact, only the underpricing equilibrium exists in this case.

for the clustering in our story, but it alone would suppress clustering rather than induce it: The free-rider problem presented by the externality tends to reduce each high-quality firm's incentive to underprice. It is the asymmetric information and hence high-quality firms' desire to signal the quality that induces each of them to underprice. That is, by increasing the benefit for low-quality firms to mimic, the externality through publicity forces a high-quality firm to underprice IPO if it wants to signal its quality and to capture the benefit of the externality. In fact, the stronger the externality (i.e., the larger the ρ), the larger the underpricing.

Besides expectations, whether a high-quality firm underprices and by how much it underprices depend also on some fundamental features of the economy. One such fundamental is the borrowing cost, which can be captured by the parameter b. When b is larger, the borrowing cost is higher and underpricing is more costly for a firm, since underpricing forces the firm to borrow. In this case, the minimum level of ρ that is necessary for inducing underpricing is higher, making underpricing less likely. The amount of underpricing is also lower. Simply put, a higher borrowing cost makes signalling more effective and so less or no underpricing is needed.

Another fundamental is the average earnings by firms in the sector. Since a decrease in R_0 reduces expected earnings of both a high-quality firm and a low-quality firm, it reduces the average expected earnings. Similar to an increase in the borrowing cost, a decrease in R_0 makes underpricing less likely and reduces the amount of underpricing if underpricing occurs. This is because a reduction in R_0 reduces the intrinsic earnings difference between a high-quality firm and a low-quality firm, which reduces the temptation for a low-quality firm to mimic and hence reduces the need for underpricing as a signalling method.

Both b and R_0 are likely to fluctuate over business cycles. Since a business downturn is likely to generate both a higher borrowing cost and a lower average earnings, the frequency of underpricing and the magnitude of underpricing are likely to be lower in business downturns than in expansions. Likewise, underpricing is likely to be more common in economies where firms have an easy access to the credit market than in economies with a difficult access.

The model is capable of producing large underpricing. In this version of the model, an underpricing firm offers the shares free of charge! That is, when the expected earning difference between a high-quality firm and a low-quality firm passes the critical level bx_L^{-1}, the amount of underpricing jumps discontinuously from zero to 100% and the offer price drops to zero. Of course, a zero offer price is unrealistic. The section below extends the model to generate a positive offer price in the underpricing equilibrium. Also, we investigate alternative assumptions about the externality and the timing structure of the signalling game.

5. Extensions

5.1. Lower Bound on Equity Financing

A firm may not be able to borrow as much as it likes. This puts a lower bound on the amount of fund that the firm must raise through IPO. Let this lower bound be $Q_b s/p$, where $Q_b \in (0, 1)$. This specification incorporates the idea that, if a firm's IPO experiences a price gain, lenders may lend more to the firm (particularly when the lender is uninformed about the firm's quality even after screening). Substituting the expression for p, the constraint $q \geq Q_b s/p$ for a high-quality firm can be written as:

$$f \geq \frac{Q_b}{R_H - (1 + b x_H^{-1})(1 - q)} \equiv LB(q). \tag{5.1}$$

With this constraint, the separating action depicted by point A in Figures $2a$ and $2b$ may no longer be feasible to a high-quality firm. A scenario is depicted in Figure 5 for the case $Q_1 > 0$, in which the best separating action is given by point B rather than A. There is IPO underpricing in this case since point B lies above the full-price curve $f = s_H(q)$. That is, since the number of shares that the firm can issue to the public is bounded from below by (5.1), a high-quality firm gets into the underpricing phase even for small differences $R_H - R_0$ that would not call for underpricing in the absence of the constraint. The underpricing signalling equilibrium exists if and only if the curve $f = LB(q)$ crosses the curve $f = IND_L(q)$ before crossing $f = S_H(q)$. Equivalently, this requires $IND_L(Q_b) > S_H(Q_b)$.

Figure 5 here.

Two properties of the separating equilibrium in this extension are in contrast with the simple model. First, an underpricing firm's offer price can be positive, as at point B in Figure 5. Second, the number of shares issued to the public by a firm does not necessarily increase with the earnings when the amount of underpricing is small. In Figure 5, for example, when R_H increases, the curve $f = IND_L(q)$ shifts up but the curve $f = LB(q)$ shifts down. These two forces change f in opposite ways and so analytically the effect of R_H on f is ambiguous in the underpricing equilibrium. When the externality is sufficiently strong, however, f is likely to increase with R_H. In this case the magnitude of underpricing is large, as in the simple model.

5.2. A Firm's Own Influence on Publicity

In the simple model we assumed that a high-quality firm benefits from other high-quality firms' IPO underpricing but not from its own underpricing. However, it is frequently suggested that a firm underprices IPO to create publicity for itself. To investigate this motive, let us return to the simple model and modify the specification of market expectations on m as

$$E(m|D, d, \text{perceived quality of the firm} = x_H) = \rho(\gamma d + D),$$

where $\gamma > 0$ is the relative impact of the firm's own underpricing on its expected earnings. The simple model examined before corresponds to $\gamma = 0$. To facilitate discussion in the current case, let us restrict $0 \leq \rho < 1/\gamma$.

Now that $R_H = R_0 x_H / x_L + \rho(\gamma d + D)$, which depends on the firm's own action, the firm cannot take R_H as given. Denote the part that the firm takes as given by $W = R_0 x_H / x_L + \rho D$. The market price of a firm under the market's belief I can be found as

$$p_I = \frac{1}{1 - I\rho\gamma} \left[IW + (1 - I)R_0 - I\rho\gamma q/f - (1 + bE_I x^{-1})(1 - q) \right]. \tag{5.2}$$

The constraint $s \leq p_H$ can be written as

$$f \geq q / \left[W - (1 + bx_H^{-1})(1 - q) \right]. \tag{5.3}$$

The following proposition shows that the qualitative results in this extended environment are similar to those in the simple model (see Appendix C for a proof):

Proposition 5.1. *There exist $\gamma_1 > 0$ and $\rho_1 \in (0, 1/(1 + \gamma))$ such that an underpricing equilibrium exists if $\rho \in (\rho_1, 1/(1 + \gamma))$ and $\gamma \leq \gamma_1$. There exist $\gamma_2 > 0$ and $\rho_2 \in (0, 1/\gamma)$ such that a no-underpricing market equilibrium exists if $0 \leq \rho < \rho_2$ and $\gamma \leq \gamma_2$. Moreover, $\rho_1 < \rho_2$ and so the two market equilibria coexist when $\rho \in (\rho_1, \rho_2)$ and $\gamma \leq \min\{\gamma_1, \gamma_2\}$.*

5.3. Sequential Decisions

The simple model illustrates the tendency for IPO underpricing to cluster if firms go to the IPO market at the same time. Of course, by assuming firms to move simultaneously we do not mean that firms in reality literally make their IPO decisions at the same date. Rather, we mean that some firms' IPO dates may be close to each other so that one firm cannot change the IPO decision to take into account of observed actions by other firms. Although this interpretation is appealing, one may still want to know what happens if firms can modify their IPO decisions upon observing other firms' actions. This we analyze now. We show that there still is a tendency for IPO underpricing to cluster in such a sequential game.

Consider only two firms, firm 1 and firm 2. Firm 1's IPO is at date 1 and firm 2's is at date 2; such timing is determined by exogenous restrictions. To simplify matters, we assume that both firms have earnings only at date 2 and there is no time discounting. Let d_i be the amount of underpricing by firm $i = 1, 2$. The market's expectations on firm i's m, given the perception that the firm is high quality, are given by (2.2) with D being replaced by $d_{i'}$ $(i' \neq i)$. The appropriate restriction on ρ in this case is $0 \leq \rho < \alpha^{-1/2}$.

Given d_1, firm 2's pricing decision is analogous to that analyzed in the simple model. That is, if firm 2 is a low-quality firm, then $d_2 = 0$; if firm 2 is a high-quality firm, then

$$d_2 = \begin{cases} 0, & \text{if } d_1 < D_0 \\ \rho d_1 + R_0 x_H/x_L - 1 - b x_H^{-1}, & \text{if } d_1 \geq D_0, \end{cases}$$

where D_0 is defined in (3.15).

For firm 1, it anticipates the influence of its pricing decision on firm 2's. Given the prior on firm 2's quality, firm 1's expectations on firm 2's amount of underpricing are

$$\alpha \chi_{(d_1 > D_0)} (\rho d_1 + R_0 x_H/x_L - 1 - b x_H^{-1}),$$

where $\chi_{(d_1 > D_0)} = 1$ if $d_1 > D_0$ and 0 otherwise. Suppose firm 1 chooses $d_1 < D_0$. Then $d_2 = 0$ and there is no publicity from which firm 1 can benefit. In this case firm 1's best decision is $d_1 = 0$ and the payoff to both firms is identical to that in the no-underpricing equilibrium in the simple case. This can be a market equilibrium in the current case if and only if the payoff to firm 1 is not lower than that generated by the action $d_1 \geq D_0$.

Now suppose firm 1 chooses $d_1 \geq D_0$. If the market believes that the firm is high-quality with probability I, then the expected earning of the firm is

$$R_I = (1 - I)R_0 + I \left[R_0 x_H/x_L + \rho\alpha(\rho d_1 + R_0 x_H/x_L - 1 - b x_H^{-1}) \right].$$

Slightly abuse notation to denote $W = (1 + \rho\alpha) R_0 x_H/x_L - \rho\alpha(1 + b x_H^{-1})$. The market price of such a firm under the belief I is

$$p_I = \frac{1}{1 - I\alpha\rho^2} \left[IW + (1 - I)R_0 - I\alpha\rho^2 q/f - (1 + bE_I x^{-1})(1 - q) \right].$$

This is similar in form to the market price in the last subsection, with $\alpha\rho^2$ replacing $\rho\gamma$, and so the offer price decision of firm 1 can be analyzed analogously to that in the proof of Proposition 5.1. The proof of the following result is in Appendix D:

Proposition 5.2. *There exist $\rho_3, \rho_4 \in (0, \alpha^{-1/2})$ such that firm 1 underprices IPO if and only if it is high-quality and $\rho \in (\rho_3, \rho_4)$. Firm 2 underprices IPO if and only if it is high-quality and if firm 1 underprices IPO.*

Example 5.3. *The interval (ρ_3, ρ_4) can be non-empty: When $\alpha = 0.1$, $b = 0.2$, $x_L = 1$, $x_H = 1.18$ and $R_0 = 1.02$, $\rho_3 = 1.75 < \rho_4 = 1.91$.*

As the example shows, there are cases in which IPO underpricing occurs. The important point is that, when firm 1 underprices IPO, firm 2 will do so as well if it is high quality. Since such underpricing would not occur if the firm were the only one in the industry or if publicity had no effect on expected earnings, the result shows the tendency of clustering in underpricing, just as in the case of simultaneous decisions. It is not surprising then that underpricing in the case of sequential moves also requires the externality to be strong enough (i.e., $\rho > \rho_3$).[15]

In contrast to the case of simultaneous moves, too strong an externality (i.e., $\rho > \rho_4$) will destroy the underpricing equilibrium in the current case. This is because firm 1 can entice high-quality firm 2 to underprice and so, loosely put, firm 1 can choose between the underpricing equilibrium and the no-underpricing equilibrium. Only when the payoff in the underpricing equilibrium is higher than without underpricing does firm 1 choose to underprice IPO. Since large underpricing is costly, an underpricing equilibrium is not always better than no-underpricing. In particular, when $\rho > \rho_4$, the amount of underpricing required to separate a high-quality firm from a low-quality one is so large that makes underpricing not worthwhile to firm 1.

Multiplicity of equilibria disappears with sequential moves. However, this may be an artifact of the exogenously fixed order of moves by the two firms. Being a first mover is costly in the current setup. Firm 1 must underprice sufficiently in order to entice firm 2 to underprice. If firms could choose when to go to the IPO market, it would be likely that they choose to go to the market at the same date. Then, in the absence of coordination the multiplicity analyzed in the simple model would reappear.[16]

6. Conclusion

In this paper we have shown that large magnitudes and the clustering of Internet IPO underpricing can be attributed to firms' desire to benefit from the industry's publicity. We assume that each firm has private information about its quality and, if perceived as high-quality, a firm's expected earnings increase with the industry's publicity that is generated by all firms' IPO underpricing. In this environment, a high-quality firm will not underprice its IPO if it expects other firms not to underprice, but it will underprice its IPO if it expects other firms to do so and if the firm's expected earnings increase sufficiently with the industry's publicity. Thus there is clustering in IPO underpricing and the magnitude of underpricing is large if the publicity is great. The industry's publicity induces a high-quality firm to take such a costly action as IPO underpricing

[15] Firm 1's underpricing is not always echoed by firm 2, since firm 2 may turn out to be a low-quality firm. This uncertainty is eliminated in the case of simultaneous moves with the assumption of a large number of firms. As a result, the amount of underpricing is larger there than here.

[16] Tambanis and Bernhardt (1999) explicitly model the possibility that firms can delay the timing of their equity issue. However, they do not analyze IPO underpricing.

because the publicity increases the temptation for a low-quality firm to mimic a high-quality firm, which makes IPO underpricing necessary for a high-quality firm to separate itself out. Private information is critical for the story: If a firm's quality were public information, the publicity would only induce free-riding and eliminate underpricing altogether.

Although our analysis is phrased for the Internet industry, the results are not confined to this industry but rather applicable to any industry where the industry's publicity is important to each good firm's expected earnings. At this general level, our results indicate that the clustering of large IPO underpricing is both fragile and specific — fragile because large underpricing is unlikely to occur if other firms in the industry are not expected to underprice; specific because important for the story is some publicity that firms collectively benefit from. Thus, the clustering of underpricing may be only a temporary phenomenon for new industries such as the Internet industry where publicity is likely to yield a large benefit initially. As the industry becomes established, competition against other firms in the same industry becomes more important than against firms in traditional sectors. In this case one firm's underpricing may hurt rather than benefit other firms in the same industry and so clustering of large IPO underpricing becomes rare.[17] Alternatively, clustering of large underpricing may disappear when there is adverse news about the new industry.[18] The fragility and specificity of the clustering of underpricing might explain why there is no strong evidence supporting a monotonic relationship between firms' IPO underpricing and expected earnings (Michaely and Shaw (1994)).

Our model also indicates that clustering of IPO underpricing is more likely to occur when the marginal borrowing cost is low than when it is high, or when the average return to firms is high than when it is low. Thus, even for established industries, more firms will underprice IPOs when the economy is in good times than in bad times. Moreover, the model suggests that there need not be a general, monotonic relationship between the fraction of the stake withheld by the firm's original owners and the firm's expected earnings, a relationship emphasized by pervious signalling models but rejected empirically by Michaely and Shaw (1994).

To conclude the paper, we note that IPO underpricing is a form of advertisement for a firm. This is true in previous signalling models, but more so in the current model since the intention to underprice here is to benefit from the industry's publicity. The question is then why a firm chooses this form of advertisement over other advertising methods. As explained by Allen and

[17] Competition among firms in the industry corresponds to the case $\rho < 0$. Since underpricing is the best response to other firms' underpricing only if $\rho D > bx_L^{-1} - R_0(x_H/x_L - 1)$ (see (3.15) and (3.16)), under Assumption $1D$ there cannot be an underpricing equilibrium when $\rho < 0$.

[18] An example is the Biotech industry that experienced large underpricing in IPOs at the beginning of the 1990s. The heat over biotech stocks cooled down considerably when the Food and Drug Administration rejected several promising drugs such as Centocor Inc.'s Centoxin, a medicine meant to fight a deadly bacteria infection common in surgery patients.

Faulhaber (1989), other advertising methods must be monitored in order for them to be credible, which may be costly or impossible for each investor. In contrast, IPO underpricing requires no monitoring and regulations require the firm to commit to the offer price. IPO underpricing also reduces the probability of lawsuits if subsequently the firm does not do well. More specific to the environment in this paper, other advertising methods may not be effective when the entire industry has just started. In contrast, large gains in share prices of firms in the industry are "hard" evidence that might convince investors about the industry's bright future.

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Appendix

A. Proofs of Lemmas 3.1 and 3.2

For Lemma 3.1, we have:

$$\begin{aligned}
V^0(x_H) &= (1-f_0)\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right] \\
&\leq \{1-q_0/\left[R_\alpha - \left(1+bE_\alpha x^{-1}\right)(1-q_0)\right]\}\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right] \\
&\leq \left\{1-q_0/\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right]\right\}\left[R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0)\right] \\
&= R_\alpha - \left(1+bx_H^{-1}\right)(1-q_0) - q_0 \\
&\leq R_\alpha - 1.
\end{aligned}$$

The first inequality follows from substituting the lower bound for f_0 in (3.5); the second inequality follows from the fact that the preceding expression is increasing in x; and the last inequality follows from the fact that the preceding expression is increasing in q_0. The same procedure establishes $V^0(x_L) \leq R_\alpha - 1$.

For Lemma 3.2, the monotone and concavity features of $S_H(q)$ and $IND_L(q)$ can be verified directly. To prove other properties stated in the lemma, note that

$$S_H(1) = 1/R_H < 1/R_\alpha < 1 - (R_\alpha - 1)/R_H < 1 - V^0(x_L)/R_H = IND_L(1).$$

The third inequality above follows from Lemma 3.1.

Consider first the case $Q_1 < 0$ (see Figure 2a). In this case the relevant range of q is $q \in [0,1]$. Since $Q_1 < 0$, we have

$$S_H(0) = 0 < 1 - V^0(x_L)/(R_H - 1 - bx_L^{-1}) = IND_L(0).$$

Thus, the curve $IND_L(q)$ lies above the curve $S_H(1)$ at both ends. If we can show that the curve $IND_L(q)$ crosses the curve $S_H(q)$ always from below if they ever cross each other in the positive quadrant, then there cannot be any crossing between the two curves, i.e., $S_H(q) < IND_L(q)$ for all $q \in [0,1]$. To show the crossing property, suppose that the two curves cross each other at $q_c \in [0,1]$, i.e.,

$$1 - V^0(x_L)/\left[R_H - (1+bx_L^{-1})(1-q_c)\right] = q_c/\left[R_H - (1+bx_H^{-1})(1-q_c)\right]. \tag{A.1}$$

Computing the derivatives $IND_L'(q)$ and $S_H'(q)$ and substituting $V^0(x_L)$ from (A.1) shows that $IND_L'(q_c) - S_H'(q_c)$ has the same sign as that of the following expression:

$$\begin{gathered}
[R_H - (1+bx_L^{-1})(1-q_c)]q_c bx_H^{-1} + \left[R_H - (1+bx_H^{-1})(1-q_c) - q_c\right] \times \\
\left\{(1+bx_L^{-1})\left[R_H - (1+bx_H^{-1})(1-q_c)\right] - \left[R_H - (1+bx_L^{-1})(1-q_c)\right])\right\}.
\end{gathered}$$

The expression in $\{.\}$ is clearly positive. Also, Assumption $1A$ implies

$$R_H - (1 + bx_H^{-1})(1 - q_c) - q_c > R_H - (1 + bx_H^{-1}) > 0.$$

Since $Q_1 < 0$, then $R_H - (1 + bx_L^{-1})(1 - q_c) > V^0(x_L) > 0$. Thus, $IND_L'(q_c) > S_H'(q_c)$. That is, the curve $IND_L(q)$ is steeper than the curve $S_H(q)$ whenever the two cross each other. This is the desired result and so $S_H(q) < IND_L(q)$ for all $q \in [0, 1]$ in this case.

Consider now the case $Q_1 > 0$. Since $IND_L(q) < 0$ and $S_H(q) > 0$ if $0 \leq q < Q_1$, the two curves cannot cross each other in this range. Thus, consider only the range $q \geq Q_1$. In this range the above proof for the crossing property between $IND_L(q)$ and $S_H(q)$ goes through. Moreover, $IND_L(Q_1) = 0 < S_H(Q_1)$. Therefore, there is a unique crossing between the two curves.

Along $f = IND_L(q)$, a high-quality firm's payoff is

$$[1 - IND_L(q)][R_H - (1 + bx_H^{-1})(1 - q)] = V^0(x_L) \cdot \frac{R_H - (1 + bx_H^{-1})(1 - q)}{R_H - (1 + bx_L^{-1})(1 - q)},$$

which is a decreasing function of q. Along $f = S_H(q)$, a high-quality firm's payoff is

$$[1 - S_H(q)] \left[R_H - (1 + bx_H^{-1})(1 - q)\right] = R_H - (1 + bx_H^{-1})(1 - q) - q,$$

which is an increasing function of q. QED

B. Proofs of Propositions 3.3 and 3.4

We locate the position of the pooling action (f_0, q_0). Since the pooling action must satisfy (3.5), it must lie on or above the curve $f = S_\alpha(q)$. Also, it can be verified that $IND_L(q_0) > f_0$ and so the point (f_0, q_0) must lie below the curve $f = IND(q)$. This implies $f_0 > f_b$ in the case $Q_1 > 0$ (see Figure $2b$).

Consider first the case $Q_1 > 0$ (Figure $2b$). Since $f_b < f_0$ in this case and $R_H > R_\alpha$, the gain to a high-quality firm from the deviation to (f_b, q_b) is strictly positive. Thus there cannot be a pooling equilibrium in this case. The only equilibrium is a separating equilibrium (f^*, q^*) defined by (3.14). The condition corresponding to this case, $Q_1 > 0$, becomes $R_H - R_0 < bx_L^{-1}$.

Now consider the case $Q_1 \leq 0$, where the separating actions are given by (3.13). These actions may not necessarily generate a higher payoff to a high-quality firm than in the pooling equilibrium. In fact, since

$$\begin{aligned}(1 - f_b)R_H - (1 - f_0)R_\alpha &= \tfrac{V^0(x_L)}{R_H - (1+bx_L^{-1})} R_H - (1 - f_0)R_\alpha \\ &= \tfrac{1 - f_0}{R_H - (1+bx_L^{-1})} \left[R_\alpha - R_H(1 - q_0)\right],\end{aligned}$$

the gain to a high-quality firm from deviating from the pooling action to (f_b, q_b) is strictly positive if and only if $q_0 > 1 - R_\alpha / R_H$. Thus, (f^*, q^*) form a unique separating equilibrium against pooling actions with q_0 sufficiently close to 1. In this case the corresponding condition ($Q_1 \leq 0$) becomes $R_H - R_0 \geq bx_L^{-1}$. This completes the proof of Proposition 3.3.

For Proposition 3.4, we know from the above that a pooling action satisfies the Cho-Kreps intuitive criterion if and only if (3.3), (3.4), (3.5), $Q_1 \leq 0$ and $q_0 \leq 1 - R_\alpha / R_H$ are all satisfied. From the definition of Q_0 in (3.4) we have $Q_0 > 0$ if and only if $R_\alpha - 1 - bE_\alpha x^{-1} < 0$, i.e., iff

$$\alpha < \alpha_0 \equiv \frac{1 + bx_L^{-1}}{R_H + b(x_L^{-1} - x_H^{-1})}.$$

Note that $\alpha_0 \in (0, 1)$ under Assumption $1A$. Consider the case $\alpha > \alpha_0$ and so $Q_0 < 0$, in which case all $q_0 \in (0, 1 - R_\alpha / R_H]$ satisfy (3.4). For any such q_0, let f_0 be such that (3.5) holds with equality and note $f_0 \in (0, 1)$. The payoff to a low-quality firm from this pooling action is

$$\left[1 - \frac{q_0}{R_\alpha - (1 + bE_\alpha x^{-1})(1 - q_0)}\right] \left[R_\alpha - (1 + bx_L^{-1})(1 - q_0)\right].$$

Both terms of the product are increasing functions of q_0 (for $q_0 > 0 > Q_0$). Thus the payoff is maximized by setting $q_0 = 1 - R_0 / R_H$. If this maximum pooling payoff satisfies (3.3) with strict inequality, then there exist q's lower than but close to q_0 that satisfy (3.3) as well. After substituting $R_\alpha = R_0 + \alpha(R_H - R_0)$ and $E_\alpha x^{-1} = x_L^{-1} - \alpha(x_L^{-1} - x_H^{-1})$, the requirement that the maximum pooling payoff satisfy (3.3) with strict inequality can be written as

$$\alpha - \frac{1 - \alpha}{R_H - 1 - bx_L^{-1} + b\alpha(x_L^{-1} - x_H^{-1})} + \frac{R_H - R_0(1 + bx_L^{-1})}{(R_H - R_0)(R_H - 1 - bx_L^{-1})} > 0.$$

The left-hand side of the above inequality is an increasing function of α. When $\alpha = 0$, its value is negative. When $\alpha = 1$, its value has the same sign as

$$(R_H - R_0)(R_H - 1 - bx_L^{-1}) + R_H - R_0(1 + bx_L^{-1}).$$

This is positive, since $R_H \geq R_0 + bx_L^{-1}$ (as $Q_1 \leq 0$) and the above expression has a value 0 when $R_H = R_0 + bx_L^{-1}$. Therefore there exists $\underline{\alpha} \in (0, 1)$ such that (3.3) is satisfied with strict inequality for the above described (f_0, q_0) if $\alpha > \underline{\alpha}$. Define $\bar{\alpha} \equiv \max\{\alpha_0, \underline{\alpha}\}$. For $\alpha > \bar{\alpha}$, there exist pooling actions (f_0, q_0) that satisfy the Cho-Kreps intuitive criterion. These actions can be supported as pooling equilibria for given R_H by the following beliefs: Any deviation from such (f_0, q_0) is believed to be coming from a low-quality firm.

On the other hand, if $\alpha < \underline{\alpha}$, no pooling action satisfies the Cho-Kreps intuitive criterion. QED.

C. Proof of Proposition 5.1

Let V_L^0 be the payoff to a low-quality from a pooling action (f_0, q_0). As in the simple model, we find separating actions that generate lower payoffs to a low-quality firm than in a pooling equilibrium. Then we choose the best among these actions as a candidate for the action of a high-quality firm in a separating equilibrium. If a low-quality firm deviates from the pooling action to an action (f, q) and is perceived as a high-quality firm, the payoff is

$$(1-f)\left[W+\rho\gamma(p_H-s)-(1+bx_L^{-1})(1-q)\right]=\frac{1-f}{1-\rho\gamma}\left[W-\rho\gamma q/f-(1+z)(1-q)\right],$$

where $W = R_0 x_H/x_L + \rho D$ and $z = \rho\gamma b/x_H + (1-\rho\gamma)b/x_L$. This payoff is less than that in the pooling equilibrium if and only if

$$q < G(f) \equiv \frac{1+z-W+(1-\rho\gamma)V_L^0/(1-f)}{1+z-\rho\gamma/f}, \text{ for } f > \frac{\rho\gamma}{1+z};$$

$$q > G(f), \text{ for } f < \frac{\rho\gamma}{1+z}.$$

Figures 6*a* and 6*b* here.

Let us divide the proof into two cases.

Case 1: $W > (1+z)[1+(1-\rho\gamma)V_L^0/(1+z-\rho\gamma)]$. This case is depicted in Figure 6*a*. Let $S_H(q)$ now denote the right-hand side of (5.3) and let its inverse be S_H^{-1}. It can be shown that there exists $\gamma_1 > 0$ such that $G(f) > S_H^{-1}(f)$ in the region $f < \rho\gamma/(1+z)$ if $\gamma \leq \gamma_1$, as depicted in Figure 6*a*. Restrict attention to $\gamma \leq \gamma_1$. In this case the relevant region is $f > \rho\gamma/(1+z)$ and the shaded area is the set of actions that yield lower payoff to a low-quality firm but may yield higher payoff to a high-quality firm than in the pooling equilibrium. The following properties can be verified for the segment of $G(f)$ with $f > \rho\gamma/(1+z)$:

(1a) $G(f) > 0$ iff $f > 1-(1-\rho\gamma)V_L^0/(W-1-z)$ (i.e., iff f is higher than point A).

(1b) $G'(f) > 0$ for all $f > 1-(1-\rho\gamma)V_L^0/(W-1-z)$.

(1c) The payoff to a high-quality firm from taking actions along $q = G(f)$ is decreasing in f.

These properties imply that, if $\gamma \leq \gamma_1$, the best deviation for a high-quality firm from a pooling equilibrium is point A in Figure 6*a*. In this case, $q = s = 0$ and there is underpricing as in the corresponding case in the simple model.

Case 2: $W < (1+z)[1+(1-\rho\gamma)V_L^0/(1+z-\rho\gamma)]$. In this case, the best deviations for a high-quality firm in the region $f < \rho\gamma/(1+z)$ lie on the curve $f = S_H(q)$ and, by property (2c) below, they are strictly dominated by the action at point A in Figure 6*b*. Thus, it suffices to consider only the region $f > \rho\gamma/(1+z)$. The curve $q = G(f)$ for $f > \rho\gamma/(1+z)$ is depicted by Figure 6*b*, where the shaded area is the set of deviations that are feasible to a firm (when perceived as a high-quality firm as a result of deviation) and that generate lower payoffs to a low-quality firm than in the pooling equilibrium. A lengthy exercise can establish the following properties, some of which are depicted in Figure 6*b*:

(2a) There exists a level $f_c \in (\rho\gamma/(1+z), 1)$ such that the curve $q = G(f)$ is decreasing in f for $f \in (\rho\gamma/(1+z), f_c)$ and increasing in f for $f \in (f_c, 1)$.

(2b) $S_H(1) = 1/W > \rho\gamma/(1+z)$ and $G(1/W) < 1$. That is, the intersection between the curve $f = S_H(q)$ and $q = 1$ lies in the region $q > G(f)$ and $f > \rho\gamma/(1+z)$. Since the curve $f = S_H(q)$ starts outside this region when q is small, there is at least one intersection between $f = S_H(q)$ and $q = G(f)$, as depicted by point A in Figure 6*b*.

(2c) The payoff to a high-quality firm from taking actions along the curve $f = S_H(q)$ is increasing in q.

(2d) The payoff to a high-quality firm from taking actions along the curve $q = G(f)$ (for $f > \rho\gamma/(1+z)$) is decreasing in f for all $f \geq (\rho\gamma/W)^{1/2}$.

(2e) There exists $\gamma_2 > 0$ such that, if $\gamma \leq \gamma_2$, then the intersection (point A) has $f \geq (\rho\gamma/W)^{1/2}$.

These properties imply that, if $\gamma \leq \gamma_2$, the payoff to a high-quality firm from deviating from the pooling action is maximized at the intersection between the curve $f = S_H(q)$ and $q = G(f)$, such as point A in Figure 6*b*. There is no underpricing in this case.

When α is sufficiently small, in both case 1 and case 2 one can also show that the payoff at point A to a high-quality firm is higher than the payoff in the pooling equilibrium, provided that the market views such deviation as coming from a high-quality firm. Thus, the action given by point A is the separating equilibrium that satisfies the Cho-Kreps criterion. Substitute $W = R_0 x_H/x_L + \rho D$ and note that the payoff to a low-quality firm is $R_0 - 1$ in the absence of pooling (thus V_L^0 in the above analysis is replaced by $R_0 - 1$). We have,

$$\begin{aligned} d \quad = \quad & p_H = \tfrac{1}{1-\rho\gamma}\left[\rho D + R_0 x_H/x_L - 1 - b/x_H\right], \\ & \text{if } R_0 \tfrac{x_H}{x_L} + \rho D > (1+z)\left[1 + \tfrac{(1-\rho\gamma)(R_0-1)}{1+z-\rho\gamma}\right], \end{aligned} \tag{C.1}$$

$$d = 0 \qquad \text{if } R_0 \frac{x_H}{x_L} + \rho D < (1+z)\left[1 + \frac{(1-\rho\gamma)(R_0-1)}{1+z-\rho\gamma}\right]. \tag{C.2}$$

To solve for market equilibria, impose symmetry $d = D$. Doing so for case 1 yields

$$d = D = \frac{R_0 x_H/x_L - 1 - b/x_H}{1 - \rho(1+\gamma)}.$$

Thus, $d > 0$ only if $\rho < 1/(1+\gamma)$. Also, (C.1) must be satisfied in order to have $D > 0$, i.e.,

$$R_0 \frac{x_H}{x_L} + \rho \frac{R_0 x_H/x_L - 1 - b/x_H}{1 - \rho(1+\gamma)} > (1+z)\left[1 + \frac{(1-\rho\gamma)(R_0-1)}{1+z-\rho\gamma}\right]. \tag{C.3}$$

Note that z and $(1-\rho\gamma)/(1+z-\rho\gamma)$ are decreasing functions of ρ and so is the right-hand side of the above inequality. The left-hand side is an increasing function of ρ. Since the inequality is satisfied for $\rho = 1/(1+\gamma)$ and violated for $\rho \to 0$, there exists a critical level $\rho_1 \in (0, 1/(1+\gamma))$ such that the above inequality is satisfied if and only if $\rho > \rho_1$. Therefore, an underpricing equilibrium exists if $\rho_1 < \rho < 1/(1+\gamma)$ and $\gamma \leq \gamma_1$.

For the no-underpricing equilibrium, impose $d = D = 0$ in case 2. The existence condition becomes

$$R_0 \frac{x_H}{x_L} < (1+z)\left[1 + \frac{(1-\rho\gamma)(R_0 - 1)}{1 + z - \rho\gamma}\right]. \tag{C.4}$$

The right-hand side of this inequality is a decreasing function of ρ. The inequality is satisfied when $\rho \to 0$ and violated when $\rho \to 1/\gamma$. Thus, there exists $\rho_2 \in (0, 1/\gamma)$ such that the inequality is satisfied for $0 < \rho < \rho_2$. If $\gamma \leq \gamma_2$, in addition, the no-underpricing equilibrium exists.

Comparing (C.3) and (C.4) immediately shows $\rho_1 < \rho_2$. Therefore, the underpricing equilibrium and the no-underpricing equilibrium coexist if $\rho \in (\rho_1, \rho_2)$ and $\gamma \leq \min\{\gamma_1, \gamma_2\}$. This completes the proof of Proposition 5.1. QED

D. Proof of Proposition 5.2

We have already argued in the text that firm 2 underprices only if firm 1 underprices sufficiently (i.e., if $d_1 \geq D_0$). Analogous to the derivation of (C.1) in Appendix C, we have:

$$d_1 = \frac{1}{1 - \alpha\rho^2}(W - 1 - b/x_H), \tag{D.1}$$

$$\text{if } W > (1 + \alpha\rho^2 b x_H^{-1} + (1-\alpha\rho^2) b x_L^{-1})\left[1 + \frac{(1-\alpha\rho^2)(R_0 - 1)}{1 + \alpha\rho^2 b x_H^{-1} + (1-\alpha\rho^2) b x_L^{-1} - \alpha\rho^2}\right], \tag{D.2}$$

where $W = (1+\rho\alpha)R_0 x_H/x_L - \rho\alpha(1 + b x_H^{-1})$. The underpricing equilibrium has $q = G(f) = 0$. With V_L^0 being set to $R_0 - 1$, $G(f) = 0$ implies:

$$f = 1 - \frac{(1-\alpha\rho^2)(R_0 - 1)}{W - \left[1 + \alpha\rho^2 b x_H^{-1} + (1-\alpha\rho^2) b x_L^{-1}\right]}. \tag{D.3}$$

For firm 1 to underprice, d_2 must also be positive and so we need $d_1 \geq D_0$, i.e.

$$W - 1 - b/x_H \geq \frac{1 - \alpha\rho^2}{\rho}\left[\frac{b}{x_L} - R_0\left(\frac{x_H}{x_L} - 1\right)\right]. \tag{D.4}$$

Note that W is an increasing function of ρ and the right-hand side of (D.2) is a decreasing of ρ. Moreover, (D.2) is satisfied when $\rho \to \alpha^{-1/2}$ and is violated when $\rho \to 0$. Then, there exists $\rho_a \in (0, \alpha^{-1/2})$ such that (D.2) is satisfied if and only if $\rho \in (\rho_a, \alpha^{-1/2})$. Similarly, there exists $\rho_b \in (0, \alpha^{-1/2})$ such that (D.4) is satisfied if and only if $\rho \in [\rho_b, \alpha^{-1/2})$. Let $\rho_3 = \max\{\rho_a, \rho_b\}$. Then both (D.2) and (D.4) are satisfied if and only if $\rho \in (\rho_3, 1/\alpha^2)$.

In addition to the requirement $\rho \in (\rho_3, \alpha^{-1/2})$, the payoff to firm 1 (when it is high-quality) must be higher with $d_1 > 0$ than with $d_1 = 0$ in order for the firm to underprice. With $d_1 = 0$, the payoff to high-quality firm 1 is

$$\begin{aligned} & (1-f^*)\left[R_0 \tfrac{x_H}{x_L} - \left(1+\tfrac{b}{x_H}\right)(1-q^*)\right] = R_0 \tfrac{x_H}{x_L} - \left(1+\tfrac{b}{x_H}\right)(1-q^*) - q^* \\ = \; & (R_0 - 1)\left[R_0 x_H/x_L - (1+b/x_H)(1-q^*)\right] / [R_0 x_H/x_L - (1+b/x_L)(1-q^*)] \end{aligned}$$

where the inequalities come from substituting the definitions of (f^*, q^*) in (3.14). When $d_1 > 0$ is given by (D.1), $q = 0$ and f is given by (D.3). The total return to shareholders is $(W - 1 - bx_H^{-1})/(1-\alpha\rho^2)$ and the payoff to high-quality firm 1 from underpricing is

$$\frac{(R_0 - 1)(W - 1 - bx_H^{-1})}{W - \left[1 + \alpha\rho^2 b x_H^{-1} + (1-\alpha\rho^2) b x_L^{-1}\right]}.$$

Substituting W and simplifying, the payoff to the firm is higher with underpricing than without if and only if

$$\frac{1-\alpha\rho^2}{1+\alpha\rho} > \frac{(1-q^*)(R_0 x_H/x_L - 1 - b/x_H)}{R_0 x_H/x_L - (1+b/x_H)(1-q^*)}.$$

There exists $\rho_4 \in (0, \alpha^{-1/2})$ such that the above condition is satisfied if and only if $0 \leq \rho < \rho_4$. The level ρ_4 is not necessarily greater than ρ_3. Only when $\rho_4 > \rho_3$ and $\rho \in (\rho_3, \rho_4)$ does high-quality firm 1 underprice IPO. QED

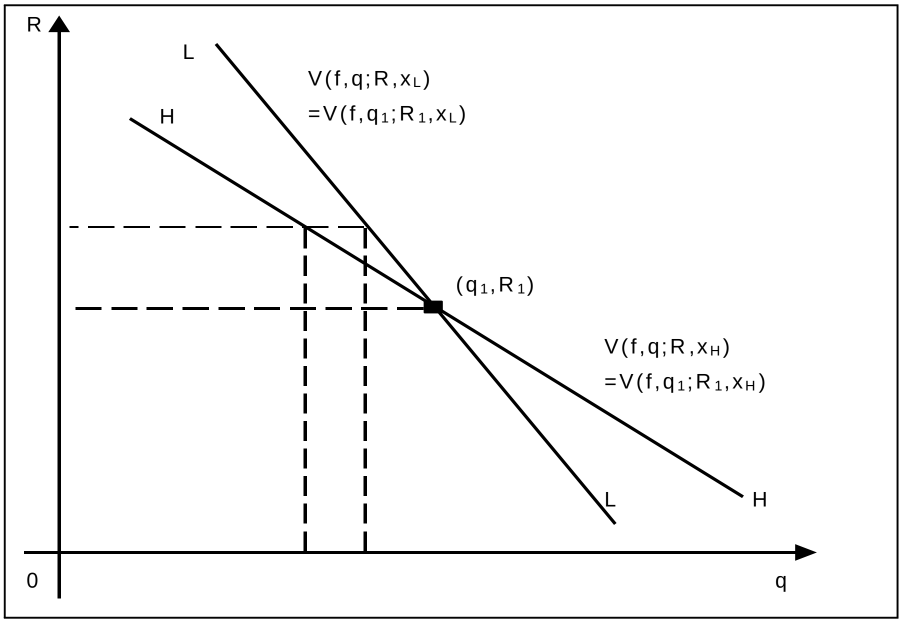

Figure 1a Firms' relative incentive to change q for fixed f
in response to an increase in R

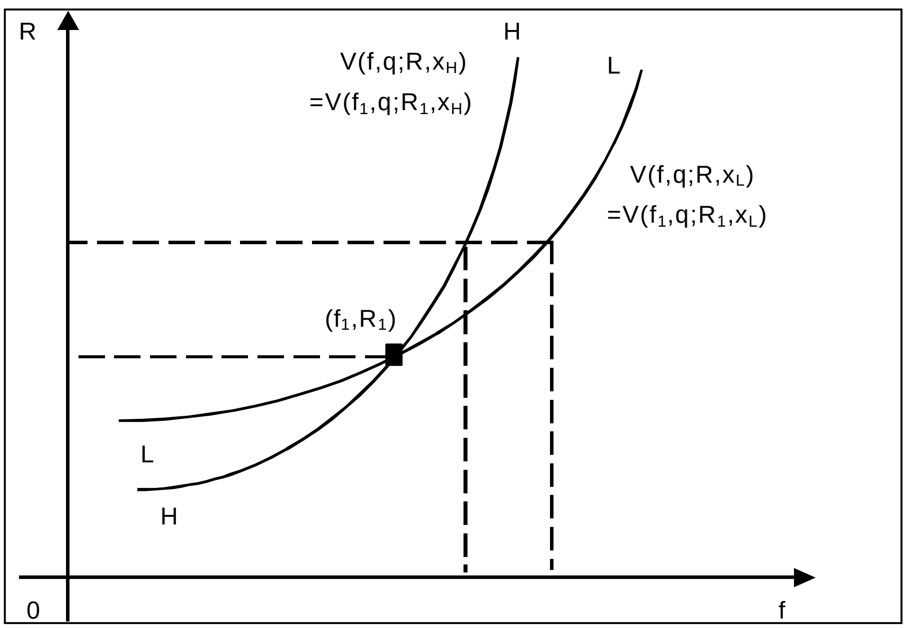

Figure 1b Firms' relative incentive to change f for fixed q
in response to an increase in R

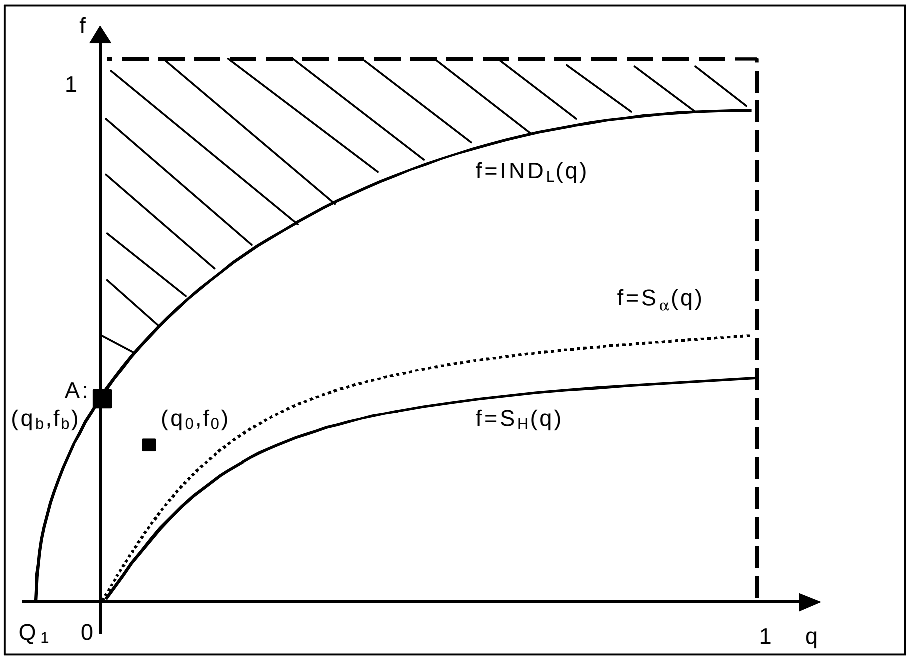

Figure 2a Deviations by a high-quality firm: $Q_1 < 0$

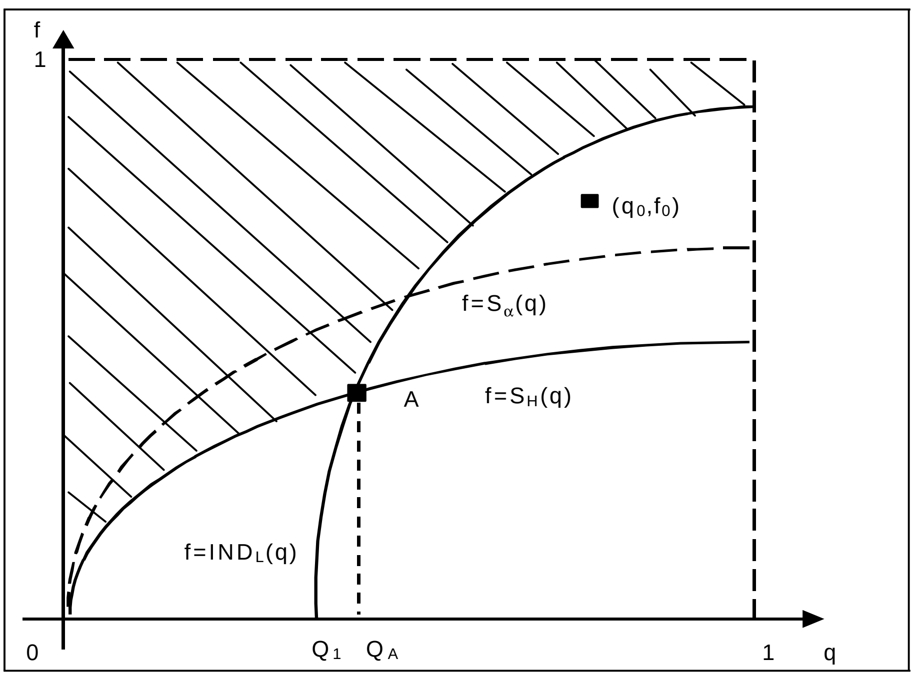

Figure 2b Deviations by a high-quality firm: $Q_1 > 0$

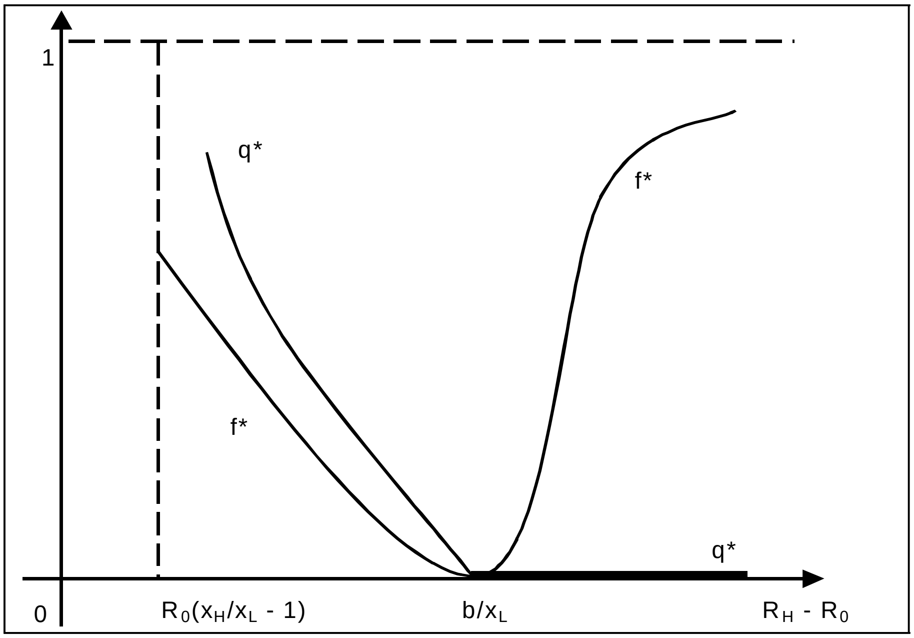

Figure 3 Dependence of (f, q) on the earnings difference between a high-quality and a low-quality firm in the separating equilibrium

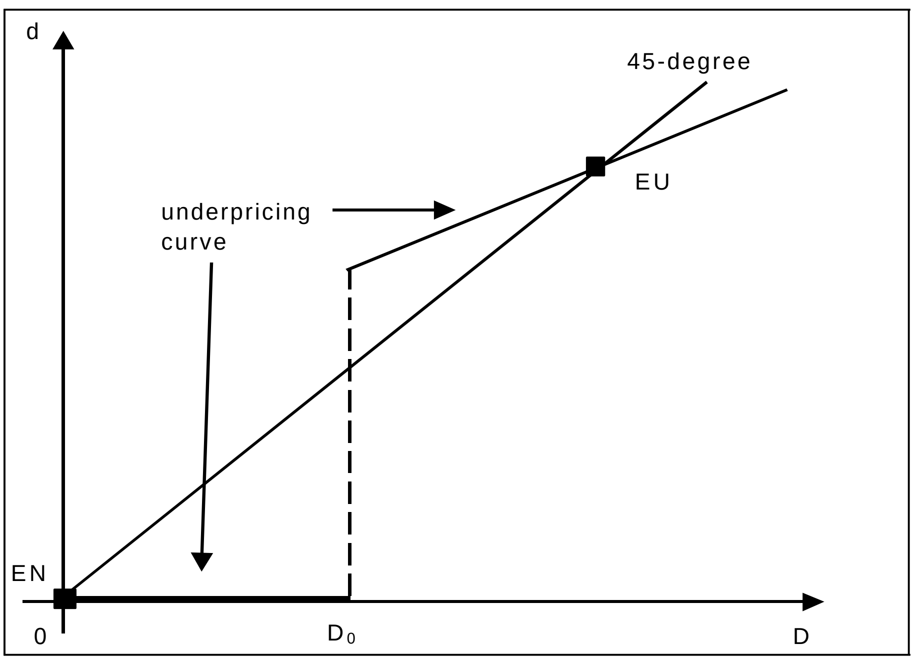

Figure 4 Market equilibria

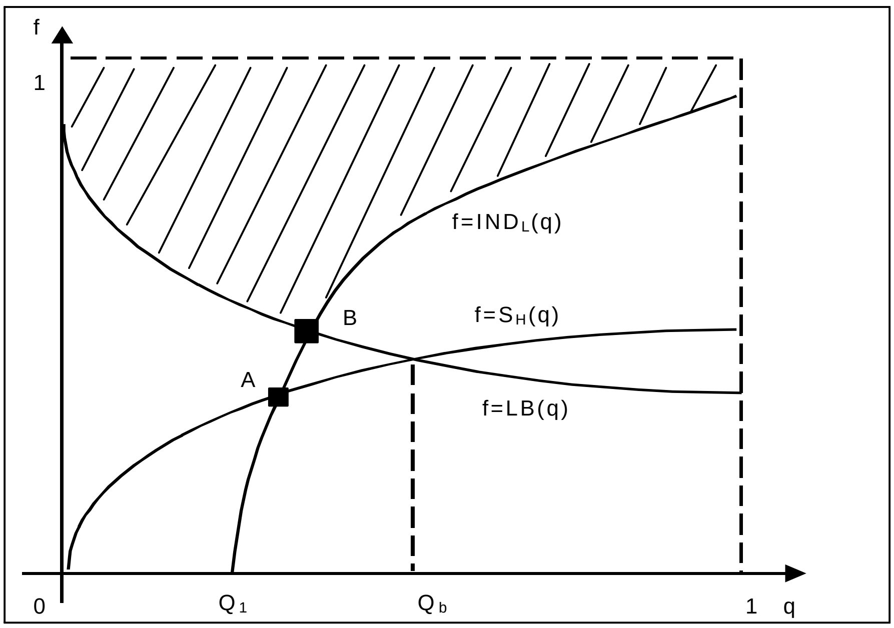

Figure 5 A separating equilibrium when there is a lower bound on the amount of equity financing

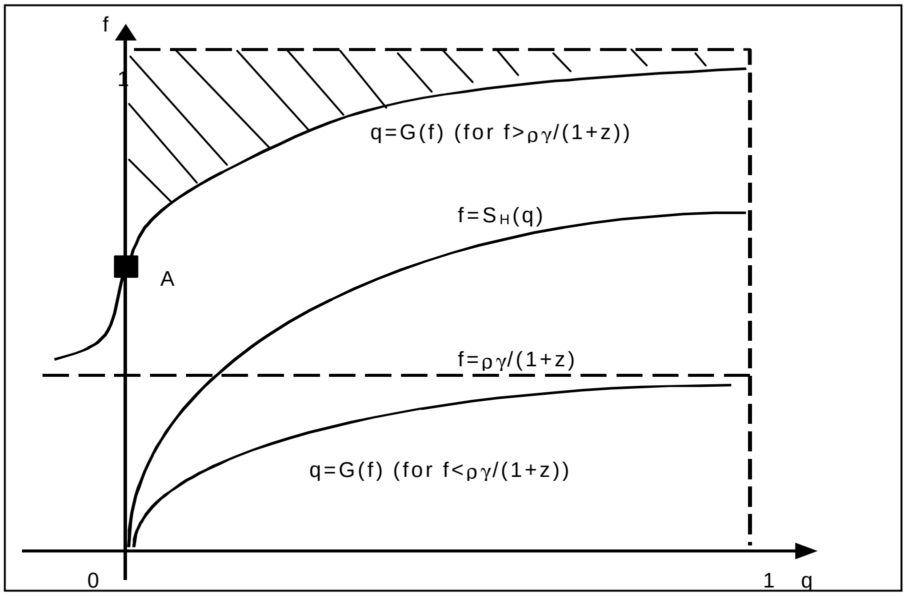

Figure 6*a* When a high-quality firm has its own influence on publicity: Case 1 (large W)

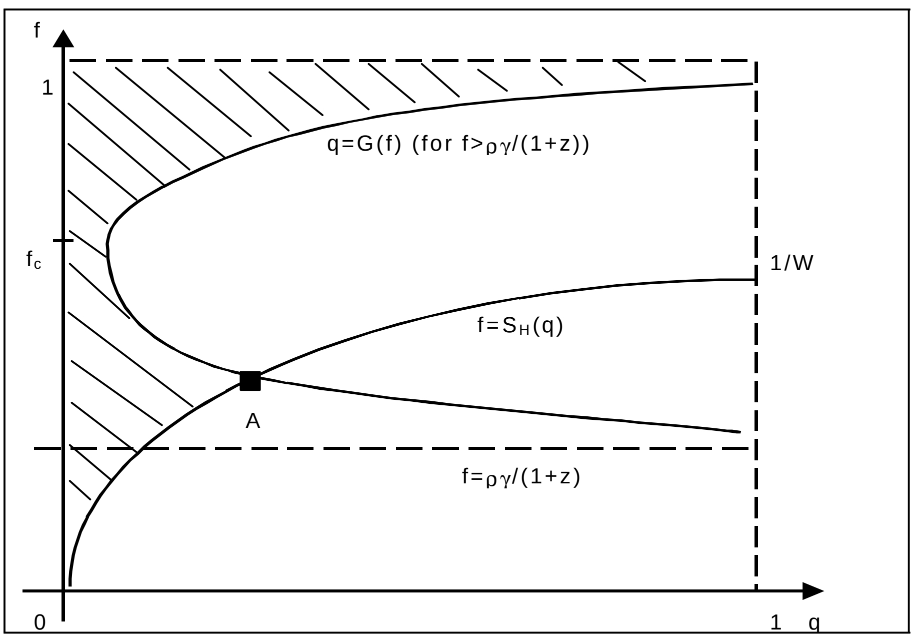

Figure 6*b* When a high-quality firm has its own influence on publicity: Case 2 (small W)